The M^5 Early Math Approach: Enhancing Math Learning in Early Childhood

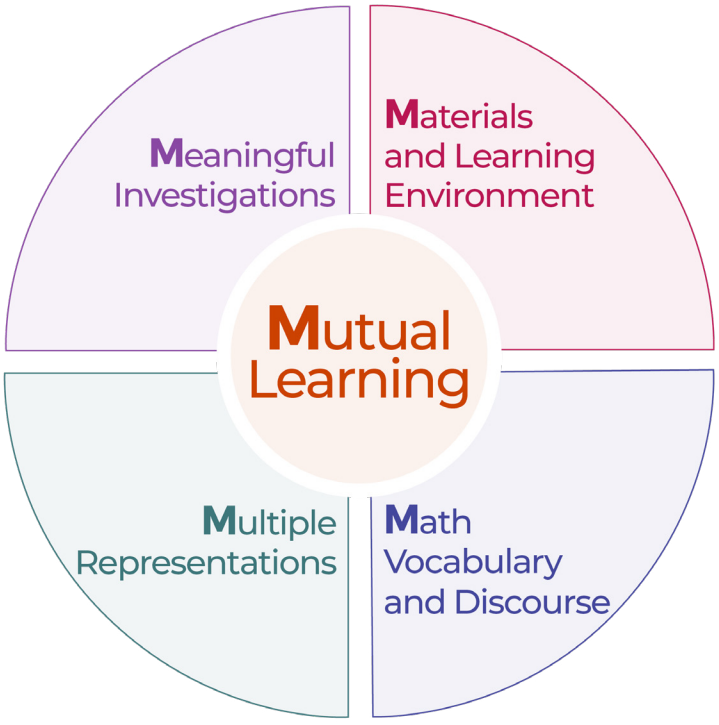
What is the M^5 Early Math Approach?

The M^5 Early Math Approach is a comprehensive, research-based approach to early math education. It includes five teaching practices to support inquiry-based and equitable learning experiences in early childhood. The method is aligned with developmentally appropriate practice[1] and California's foundations,[2,3] standards,[4] and frameworks for mathematics.[5,6] It forms an effective approach to early math education for children from birth to age eight.

The M^5 Early Math Approach includes the following five teaching practices:

- Mutual Learning
- Meaningful Investigations
- Materials and Learning Environment
- Math Vocabulary and Discourse
- Multiple Representations

The approach is called M^5 because each of these five teaching practices begins with the letter "M." This brief provides an overview of the key characteristics of the M^5 Early Math Approach (also called M^5). It describes each M^5 practice and highlights related research. Then, the brief includes suggestions on how to use M^5 practices in early learning settings. Although each of the five practices is discussed individually, many parts of these practices overlap. Early educators use M^5 practices at the same time.



Key characteristics of the M^5 Early Math Approach

Research-based. The approach is aligned with research on children's developmental progressions in early math. The M^5 practices reflect what research has shown to improve children's math learning and development.[7,8,9]

Child-centered. Children learn best when educators focus on their interests, strengths, and needs. The M^5 approach helps educators honor children's unique ways of being, including their cultures, languages, lived experiences, and abilities. Keeping the child at the center allows educators to offer learning experiences that support every child to learn, develop, and find the joy in math. By "every child," the approach means to include children of any background, race, culture, ethnicity, gender, ability, or socioeconomic status.

Inquiry-based. "Inquiry-based" means that children develop math knowledge and skills by investigating meaningful problems and authentic questions. Allowing children to learn and use math for a purpose that is related to their lives can deepen understanding of math concepts.

Flexibly applied. The M^5 approach can be used flexibly in different learning settings, with different curricula, and throughout the day with children of all ages and abilities. It encourages educators to tailor learning experiences to meet the needs, cultures, and interests of the children in their classrooms. They do this in ways that build deep and meaningful understanding of math concepts.

Mutual Learning

What is mutual learning?

Mutual learning emphasizes that learning is bidirectional. Educators learn from and are responsive to individual children, and children learn from educators and from each other. Mutual learning begins with trusting, respectful relationships among educators, children, and families.

Through intentional observations of children, and interactions with families, educators learn about children's interests, languages, cultures, lived experiences, abilities, and emerging knowledge and skills. Educators use this information to create learning environments and experiences that are meaningful to individual children. In early math, this information includes the following:

- How individual children approach mathematical problems
- What interests them
- How they show their thinking
- In what ways they experience math in their homes

How does mutual learning support children's early math learning?

Mutual learning encourages educators to recognize the uniqueness of each child and adjust math learning experiences accordingly. This practice can enhance children's engagement and achievement in mathematics and other learning domains.[10,11,12] In an environment that is inclusive and meaningful to individual children, educators do the following:

- Value diversity
- Integrate children's interests
- Foster collaboration among educators, families, and children

Mutual learning promotes inclusion. This practice helps educators create a space where all children feel valued and represented as math learners. When educators use what they learn about children to support math learning, they can promote children's sense of belonging, enhance self-esteem, and support positive identity development.[13] Through mutual learning, educators come to understand the various strengths, needs, and abilities of children. They can identify effective approaches for children who have different needs, such as the following approaches:

- Using gestures to communicate
- Offering multiple means of engagement, action, and expression
- Modifying the environment for individual abilities

Mutual learning fosters equity. Using this practice, educators can help all children view themselves as competent and capable math learners. This belief in one's own potential is especially important for children of certain genders, races and ethnicities, abilities, and other backgrounds that are underrepresented in STEM (science, technology, engineering, and mathematics) careers. Children's motivation and success increase when they can see themselves, their interests, and cultures within the curriculum and the learning environment.[7, 10]

Educators can learn about children's preferences and interests by observing children at play and during interactions and by building strong relationships with families.

Mutual Learning is key to collaborative relationships between educators, families, and children. Through mutual learning, educators can learn about families' funds of knowledge—the unique knowledge base of individual families that is relevant to their daily lives, cultures, and lived experiences.[14] They use this information to enhance math learning for children. Engaging in mutual learning with families also enhances families' engagement in children's learning and supports children's math development.[15]

How can educators engage in mutual learning?

Learn about children and their families.

Observe. Take time to carefully observe children during interactions and activities. Notice unique strengths, how children prefer to play, toys or objects they like, or challenges they experience throughout the day.

For example, notice how infants and toddlers interact with a new object. Talk to children socially to learn about their lived experiences and cultures. Or, ask children who communicate by speaking, or in other ways, about what they did over the weekend or what they enjoy playing with at home.

Observe how children use language—both their home language and English—as they explore mathematical concepts. Multilingual children might demonstrate understanding of mathematical concepts in their home language, or through actions or gestures, which educators might build on. Actively observing and being curious can provide valuable insight into children's relevant characteristics, such as the following:

- Math knowledge and skills
- Strengths and needs
- Unique cultures and lived experiences
- Interests
- Languages
- Abilities

Provide opportunities for choice and autonomy. Allow children to make choices, create their own plans, and take on leadership roles. For example, notice how young toddlers experiment with different ways to fit an object into a particular space. Or, for older children, notice their questions about the school garden and encourage them to make plans for answering those questions—"How might we figure out which plant grew more?" Taking a child-centered approach allows children to express what they know and how they think.

Partner with families. Learn about families' daily lives, cultures, and typical routines. Educators can gather information about families in many ways, such as the following:

- Communicate with families casually during drop-off and pick-up. Social conversations can help educators learn about children's cultures and lived experiences.
- Invite families into the learning setting to share information on a specific topic. For example, a family member might share a cooking experience, how to build or make something, show children how to garden, or share with children information about their jobs.
- Encourage families to share materials from home. For example, ask families to share a unique measuring tool they use at home or send in recyclable materials such as boxes and containers to add into areas of the learning setting (for example, housekeeping, block, and art areas).
- Use activities to learn about children's experiences at home. Activities might include simple handouts, in the families' preferred languages, that encourage sharing about a child's lived experiences. For example, a child and caregiver might be invited to draw differently shaped objects that they find in their home. Another activity might be a question of the day for families to respond to at drop-off, at pick-up, or through an app-based communication tool. For example, educators might ask "How did you get to school this morning?"

Use what is learned about children and families to enhance math experiences.

Support individual differences. Recognize and appreciate the individuality of each child. Gaining an understanding of the unique strengths, needs, and preferences of each child allows educators to adjust their approach and maximize learning and engagement. Children who speak more than one language, for example, might understand a math concept in their home language yet need additional support to communicate similar ideas in English. Educators might encourage the use of the home language, gestures, visuals, or other forms of communicating to help the child meaningfully engage with math.

Educators can also use mutual learning to support the individual needs of children with disabilities. They might notice and leverage children's unique strengths and preferences to support math learning.

For example, some children might prefer to express their ideas through speaking, as opposed to writing. Or, a child with an attention disorder may enjoy experiences involving music or movement yet have a difficult time focusing during seated activities. Educators might design math experiences that foster movement and integrate music. Such an approach could allow this child to thrive and more fully demonstrate their mathematical understanding.

Create a learning environment and experiences that reflect children's interests, languages, cultures, and lived experiences. For example,

- Invite children to plan and build familiar structures or buildings they notice in their community.
- Use counting songs in children's home languages.
- Offer children activities relevant to their home culture and lived experiences. For example, a bamboo steaming basket might be meaningful for children if their caregiver uses one in their home. An educator might include a steaming basket to support children's spatial thinking—exploring how to put objects in and take them out.
- Incorporate children's interests into math learning. For example, if a child is interested in throwing a ball in a basket, use this interest as an opportunity to practice counting. Count together the number of times the ball went inside the basket.

Mutual learning helps educators create an environment wherein children's individuality is respected; their interests, culture, and languages are integrated; their learning needs are met; and their ideas are valued.

Meaningful Math Investigations

What are meaningful math investigations?

Meaningful investigations are inquiry-based, playful, open-ended learning experiences that allow children to question, experiment, and use math to solve authentic problems of interest. An investigation is a learning experience wherein children are encouraged to question, experiment, gather and evaluate evidence, and draw conclusions based on the evidence they collect.[16]

These investigations are most *meaningful* when the experience is relevant to their lives. This means that the answer, or solution, is interesting to children and might be related to their real-world experiences.

For example, a meaningful investigation might involve experimenting with blocks of different sizes and shapes to figure out how to build a structure that looks like a child's apartment building. Through their problem-solving process,

children can build new knowledge, deepen existing understandings, resolve misconceptions, and discover new wonderings that motivate them to learn more.

In an investigation, children actively lead their own learning. Educators act as guides. They help focus children's attention on mathematical concepts, ask questions to encourage critical thinking, and provide information and suggestions to scaffold children's learning.

Meaningful math investigations encourage children to create their own questions and seek answers rather than passively receiving information from educators or practicing rote skills. With a math-focused lens, educators can use meaningful investigations to enhance children's understanding of math concepts.

Examples of Meaningful Math Investigations in Action

Toddlers

At the sensory table, toddlers explore math concepts as they pour water into different containers. With adult support, they might wonder "Which one has more?" or "How many cups of water fit in one container?" Through these inquiries, they might develop understandings of concepts related to big, small, more, less, inside, and outside.

Preschoolers

As preschoolers build structures in the block area, they deepen their understanding of geometry concepts. For example, they might wonder "Which block would fit in between?" or "Why does the cylinder keep falling down?" Through their attempts to construct a stable structure, or one that resembles a familiar building in their neighborhood, they might develop an understanding of three-dimensional shapes, mental rotations, and spatial vocabulary.

Early Elementary

Children in the elementary grades experience math while designing garden beds for their school garden. They explore the perimeter of different garden bed designs or determine how many plants might fit inside one bed.

In these examples, children are developing and using math knowledge for a purpose that is meaningful to them.

Why are meaningful math investigations important?

Meaningful investigations support engagement in and motivation to do math. Investigations often begin with children's questions or wonderings, which supports children's curiosity and motivation to seek answers and discover new knowledge. This motivation is intrinsic—coming from the enjoyment or satisfaction of engaging in the activity rather than an expectation of a reward or particular outcome. When math investigations are based in children's real-world experiences and have a purpose beyond learning math facts, children are more motivated and engaged in their learning.[17]

Meaningful investigations support the development of important approaches to learning skills relevant to all domains of learning, such as curiosity, engagement, and persistence. Similarly, meaningful investigations can also foster the development of a growth mindset. A growth mindset is the belief that abilities can be improved through effort, learning, and persistence.

Investigations provide children opportunities to experiment with different approaches to solving mathematical problems and to learn from both successes and failures. This process helps children develop a growth mindset, which in turn can support positive math learning.[18]

Meaningful investigations support children to collaborate, communicate, and think critically and creatively.[19] These four skills are called 21st century thinking skills—the skills that are essential for success in today's rapidly changing world. When engaged in meaningful investigations, children learn to think analytically. They evaluate evidence, make connections, and draw conclusions based on their findings.

The open nature of investigations allows children to be creative when exploring a variety of ways to solve a problem and evaluate each other's perspectives. Math investigations can be explored individually or in collaborative groups. Working collaboratively allows children to learn

from each other—a strategy that is particularly helpful when planning to support children with different needs. Working collaboratively also promotes social learning (such as turn-taking, valuing each other's strengths, and respectfully disagreeing) that can bring children together as a community of learners.[20]

Meaningful math investigations deepen children's understanding of math concepts and their ability to apply knowledge across contexts.[8,21] Investigations provide children opportunities to construct their own knowledge and apply it to real-world contexts.[22] When children are involved in the learning process, the knowledge acquired becomes more meaningful and memorable.[19, 20]

Meaningful math investigations create opportunities to *use* math in authentic ways, which enhances understanding of math concepts and children's investment in learning math. For example, children might explore ways to subtract in the context of answering a meaningful question, such as "How much has the bean plant grown since last week?" The desire to answer a meaningful question provides a purpose for using math. The open nature of the problem offers children flexibility in the strategies they use and thinking processes they apply.

How can educators create meaningful math investigations?

Pose questions and prompts that invite children to apply their math knowledge and skills to questions or problems of interest. Frame investigations as intriguing questions or problems that spark children's curiosity and interest. For example, use simple phrases like "I wonder..." or "How might we..." These open-ended phrases provide children opportunity to experiment with different approaches and strategies to solving a problem or answering a question.

Children who are nonspeaking use different ways to ask questions and wonder. Through careful observations, educators can notice curious expressions, persistent trial and error, designing and redesigning, or moments of frustration. These observations may be cues that a child is curious about something or trying to solve a problem. Educators can use strategies like narration (parallel talk) to describe what children are doing or what they might be thinking. Describing their observations shows children that their wonderings are understood and valued.

Incorporate real-world context. Consider situations happening around the learning program or school, such as the following:

- Collecting leaves outside and observing their shapes
- Building a new playground or a garden and needing to measure the space
- Preparing for snack time and thinking about how many apples are needed

Other real-world contexts might connect to children's home lives, such as the following:

- Welcoming a new baby and considering what size clothes they might need
- Making dumplings and measuring quantities of ingredients
- Playing during bath time and exploring the different sizes of cups

Provide hands-on experiences. Include physical objects to make investigations tangible and experiential. Hands-on experiences enhance engagement and understanding. Consider providing open-ended materials, or materials that can be used in a variety of ways.

It is also important to adjust materials and experiences to support children who may have physical limitations in how they can manipulate objects. For example, use lighter or smaller objects or support children to have tangible experiences with assistance. Refer to the section on materials and learning environment for more information related to open-ended materials.

Offer opportunities to investigate multiple solutions to a problem. Allow for open-ended math investigations. Open-ended investigations invite multiple approaches to solving a problem and, sometimes, more than one solution.

Open-ended investigations give children freedom to think creatively and approach problems in different ways. For example, children might investigate different ways to compare the amount of blocks used for building—some might count individual blocks while others line up the blocks to find out which line of blocks is longer.

Older children might explore different ways to use various coins to make $0.52. Some children might use two quarters and two pennies while others might use five dimes and two pennies.

Educators can acknowledge and appreciate different approaches and solutions to math investigations. For example, educators might describe the way different children approach a problem, highlighting the value and reason for their choices. Or educators might encourage children to explore other approaches (for example, "What might happen if you...?").

Emphasize reflection and explanation. Encourage children to reflect on their findings, reasoning, and problem-solving strategies. Reflection helps children process and make sense of what they learned during an investigation. Taking time to reflect and discuss an investigation helps children focus on the purpose of an experience.

Reflection also provides opportunities for educators to informally assess children's understanding. Use open-ended prompts to encourage children to reflect and explain. For example, "Tell me about the problem you were working on today" or "I noticed you working hard to figure out.... Tell me about it. How did you finally solve the problem?"

During choice time, a group of preschoolers tries to figure out how to make a ball roll down a ramp and land in a basket. Later, the educator asks the group to tell her about their time working with the ramps and balls. The children explain the following ways they tried to make the ball land in the basket:

- They changed the ramp's position by placing blocks beneath it.
- They created tunnels to stop the ball from rolling off the ramp.
- They added longer or shorter ramps to change the speed of the balls.

The educator summarizes these ideas with the children, highlighting the math concepts they explored related to size, shape, weight, distance, and speed. For example, the educator summarizes, "So you figured out that changing the position of the ramp changed the distance the ball would travel."

Materials and Learning Environment

What materials and learning environments support early math?

Math is everywhere! The environment and materials found within it provide opportunities for children to engage in math thinking.

For example, when children are outside, they might notice many small green leaves and only a few large brown leaves—demonstrating an understanding of quantity. During breakfast, children might observe the triangular shapes in their orange slice—helping them develop early geometry concepts. Or, in a dramatic play space, children might notice that they need to make an additional crib for their baby dolls so that *each* doll has a place to sleep—showing early development of algebraic thinking.

Materials and the learning environment include the materials (the objects and tools) available to children and the way the learning environment is arranged.

Everyday items all around us and in nature, such as:

- Shells, pinecones, leaves, rocks
- Buttons, plates, cups
- Toys, playground equipment
- Items that represent familiar materials from different homes

Games:

- Puzzles
- Matching games
- Board games

Math manipulatives:

- Counting chips
- Base ten blocks
- Rekenreks, or number racks

Tools:

- Balance scales
- Rulers
- Tape measures

Learning environment:

- Arrangement of furniture
- Available space
- Play equipment

Materials and the learning environment can both play a role in supporting children's development of mathematical thinking.

Why are materials and the learning environment important?

Children learn through actively interacting with the environment and playing with objects.[23,24,25] Studies show relationships between materials and children's learning outcomes in domains including language, literacy, science, social and

emotional skills, approaches to learning, and early math.[24,26]

For example, holding and playing with objects in infancy supports infants' spatial thinking skills.[27] Research suggests that materials, like board games and blocks, relate to children's math achievement.[28,29,30] Even engagement with loose parts and everyday items can enhance math learning.[23]

How can educators create a math-rich environment?

Enhance math within learning areas. Add materials to existing learning areas to promote the development of math knowledge and skills.

- Sensory table: Add variously sized buckets, scoops, and measuring tools to encourage children to explore measurement concepts such as size, capacity, more, and less.
- Art and writing area: Include stickers, pom-poms, buttons, and other loose parts that encourage children to sort, count, and develop patterns. Adding numbers (for example, number stickers, paper number cutouts, and number stencils) for children to use in open-ended ways can build their understanding of numerals. Include rulers to provide opportunities for children to explore measurement tools.
- Dramatic play area: Include differently sized plates, cups, and dress-up clothes to invite children to explore and compare differences in sizes and shapes. Offer measurement tools (for example, measuring tapes, scales, measuring cups, tablespoons, teaspoons) to encourage children to explore mathematical concepts related to measurement and quantity through pretend play.
- Building area: Provide blocks of different shapes and sizes to support children to explore various math concepts (for example, geometric concepts, spatial thinking, number, and counting).

Provide open-ended math materials. Provide open-ended materials that encourage mathematical exploration and problem-solving, such as loose parts (buttons, beads, sticks), blocks, and art materials can encourage mathematical exploration and problem-solving.. These materials allow children to engage in creative and open-ended mathematical play.

Although "closed-ended" materials (for example, puzzles and shape sorters) play an important role in children's learning, open-ended materials allow children to make decisions about how to use them. For example, providing children with a basket of variously sized and shaped rocks may encourage them to compare the rocks based on different attributes like size, weight, texture, or color. These early comparisons of attributes serve as the building blocks for math learning and may even motivate children to use measurement to learn more about how objects are different.

Open-ended materials also help children "work at levels that are appropriately challenging for them."[31] For example, children may build

structures varying in levels of complexity, or count small or big collections of objects based on their current level.

Furthermore, open-ended materials provide a context for educators to purposefully observe children to learn about their strengths, challenges, and different approaches to problem-solving. Refer to the section on mutual learning to learn more about purposefully observing children as a way to provide meaningful materials and learning environments.

Use math manipulatives and games. Provide a variety of math manipulatives such as blocks of different sizes and shapes, pattern blocks, base ten blocks, and ten-frames. These objects allow children to explore mathematical concepts through concrete experiences that promote active learning. Offer games with a math focus, for example, board games, card games, or games with dominoes or dice. These games can provide opportunities to use math concepts in playful and engaging ways.

Digital math games and educational applications. Computer and app-based math games that promote meaningful and socially interactive play can support math learning.[32]

Math through literature and storytelling. Offer and use math-related books or stories that highlight mathematical concepts and problem-solving. Math can be identified within stories and books, even if they are not intentionally focused on early math! Consider *Goldilocks and the Three Bears*. Goldilocks notices and compares different attributes throughout the story—she explores the concept of measurement.

Other stories and books might intentionally focus on math and include specific illustrations and text to draw children's attention to mathematical language and concepts. *One Is a Piñata: A Book of Numbers* by Roseanne Greenfield Thong and John Parra is an example of such a book.

As educators share stories and books with children, they can intentionally promote math vocabulary and concepts. They might pause and wonder with children about relevant shapes, measurements, counting, more and less, and other math concepts related to the stories and books.

Consider inviting families to share stories unique to their cultures as a way to connect math learning to children's families and lived experiences. Visit Count Play Explore's book section to find "Book Guides" that describe math learning in a variety of books along with activities to bring math thinking to life.

Partnering with families. Invite families to share materials from home and integrate these materials into learning experiences and learning centers. Some materials might include stories and books, tools, puzzles, or everyday objects. Children will be excited to see their lived experiences reflected in the learning setting.

In addition, partnering with families builds stronger connections between home and school and helps caregivers feel valued and included in the learning setting. Including tools or products from home, such as empty food boxes, spice containers, or a tortilla press, can invite conversations and connections between home and school.

Adapt materials and learning environments to support all learners to fully engage. It is especially important that educators provide materials and environments that support children with disabilities to fully engage. These supports might include the following:

- Assistive technology (for example, text-to-speech software)

- Tactile manipulatives (for example, base ten blocks)
- More space in a setting to allow for ease of movement for children that may use mobility aids (for example, crutches or wheelchairs)

It is important to consider children's Individual Education Programs or Individualized Family Service Plans. Educators should also collaborate with specialists providing services and seek support from administrators to acquire the necessary materials or design appropriate environmental conditions.

Math Vocabulary and Discourse

What is math vocabulary and discourse?

From a young age, children communicate about math-related ideas or concepts during play, daily routines, and interactions. They might discuss the different sizes of their dolls, compare the number of items in their pretend shopping bags, or negotiate with their caregiver about how many more bites of broccoli they need to eat during

lunch. These examples show how children use language to reason with and build understanding of math.

Math vocabulary—the words to describe math-related concepts such as number, size, quantity, or shapes—plays a significant role in children's math learning. *Math discourse* is the communication, discussion, and exchange of ideas related to math concepts and mathematical reasoning. As children gain more math vocabulary and are provided opportunities to engage in math discourse, they can communicate about math ideas and reason more accurately.

Learning math vocabulary is critical to learning mathematics.

The following examples of math vocabulary are relevant to early childhood math learning:

Numbers

One, two, three

Number operations

Add, combine, altogether, take away, subtract, group, divide

Comparisons

Compare, same, more, less, greater, heavy, heavier, tall, tallest, empty, full

Measurement

Long, longer, tall, taller, short, shorter, bigger, biggest, smallest

Two- and three-dimensional shapes and their attributes

Square, rectangle, cube, sphere, pyramid, face, edge, vertex

Spatial relationships such as position, direction, and distance in space

On, under, above, below, behind, far, next to

For example, children may use math vocabulary and discourse when discussing how to share a cookie so friends receive equal parts:

> Child A: You should cut it here.
>
> Child B: If I cut it there, it's not fair—you get more. I need to cut it in half so we have the same.

Children might also use math discourse to explain how they solved a problem:

> Child: I got six.

Educator: How do you know it is six? Can you explain how you got that answer?

> Child: Because there are two, and two, and two. And, when I count them all, it is 1, 2, 3, 4, 5, 6!

Math vocabulary and discourse can be communicated in different ways—including gestures, drawings, and manipulating physical materials. Nonspeaking discourse might include communicating through gestures. For example, when exploring plants in the garden, an educator might ask "How are they different?" A child might reach their arms above their head to show that one plant is taller than the other. Or, a child might demonstrate how they subtracted by physically moving objects from a larger set.

Very young children might communicate without speaking as well. For example, a young toddler might communicate about position or location by pointing. Other nonspeaking discourse might pertain to children with disabilities. For example, some children might use assistive technologies.

Math vocabulary and discourse can also happen in any language. Languages include English, varieties of English, and children's home language. Math knowledge and skills are transferable across languages—children might apply their mathematical understanding that they learned in one language to situations based in a different language.

For example, a child may be able to use their knowledge of counting in Spanish to support the development of their counting in English. Using children's home language helps children make connections and use all their knowledge and skills across their languages. Encouraging the use of home languages can help children feel a sense of belonging, which enhances their identities as competent math learners.

Why are math vocabulary and discourse important?

The use of math language supports children's math understanding. Research suggests that children who are exposed to rich math language have a better understanding of mathematical terms and concepts, leading to improved math achievement.[33,34] When educators use math vocabulary effectively, children can make connections between mathematical ideas and deepen their understanding of math concepts.[19,35,36]

For example, when children learn the word "cube," they may notice more cubes in their environment and begin to distinguish cubes from other shapes (for example, squares or rectangular prisms).

The use of math language and discourse is not restricted to older children. Using math vocabulary and engaging in math discourse with

infants and toddlers is important, too. Children understand language before they can use spoken words. Studies show that children who are exposed to math language in infancy tend to have stronger math achievement in preschool compared to children who are not exposed to math language early in their lives.[37]

How to promote the use of math language and discourse

The strategies below offer ideas on how to enhance math vocabulary and discourse in learning settings. All the strategies provided can be used in English and children's home languages.

Create a safe and supportive environment. Foster a culture in the learning environment where all children feel comfortable sharing their thinking without fear of judgment (refer to the section on mutual learning for more information related to promoting a sense of belonging). Emphasize that there are multiple ways to approach and solve problems and that mistakes are valuable for learning.

One way is to talk about one's own mistakes openly so children can observe this process. For example, an educator might say "Oops! I counted wrong. Let's try again." Or "Hmm... I thought this would be the right size, but I was wrong. I need a larger container." When educators create spaces where mistakes are part of a learning process, children may feel more comfortable engaging in math discourse. They know their emerging ideas will be accepted.

Encourage children's use of the home language. Children from diverse cultures and backgrounds should be encouraged to communicate about math in ways that are comfortable and familiar to them. This communication might include the use of dialects of English, informal language about math concepts, American Sign Language, or other languages and forms of communicating.

Provide multilingual learners opportunities to engage in math conversations and use math vocabulary in their home language, along with English, whenever possible.[38,39] Using a child's home language honors their culture and can help children develop feelings of belonging. It also broadens the connections children can make and enhances their ability to understand.

Pose open-ended questions. Ask open-ended questions that encourage children to think deeply about mathematical concepts, make connections, and provide explanations. Closed-ended questions like "Is it a square?" or "Which one has more?" or "Do you have four cookies?" only require children to respond with one word that is either right or wrong.

It is important to ask nonspeaking children open-ended questions, too! Instead of expecting a spoken response, educators might notice other cues that communicate a child's explanation, misconception, or sense-making process.

Although closed-ended questions serve a role in supporting children's math learning, they can limit children's engagement in discourse. Open-ended questions encourage children to justify their reasoning, engage in higher-order thinking, and use more language in general.

For example, asking "How might we figure out which one has more?" or "How do you know it's a square?" or "Why did you group the cookies like this?" will invite children to think about and explain a process. Encourage children to explain how they came to a certain conclusion and challenge them to consider other viewpoints. For example, an educator might ask "What makes you think that?" or "How do you know?" or "Why won't it work if I turn it the other way?"

Promote math conversations. Encourage children to discuss their mathematical ideas, strategies, and solutions. Consider using language frames to help children use language effectively when communicating with peers. For example, educators might invite children to use the language frame, "This group has [more/less] than that group. I think this is because [reason]."

Using language frames might be an especially useful strategy for multilingual learners.

For older children, use protocols in the classroom that facilitate peer-to-peer conversations, such as Think-Pair-Share. This routine provides children independent time to think and then share their thinking with a partner. Providing children frequent opportunities to explain their thinking to peers is an effective way to promote math vocabulary, discourse, and meaningful learning.

Make math apparent. Make math moments apparent to children throughout the day and in various settings by modeling math talk and math thinking. Narrate what children are doing, labeling the math that is relevant to their experiences.

Outdoor play

- "They went inside the tunnel!"
- "To play this game, let's divide the balls into groups of two."

Mealtime

- "There's a triangle on my plate. My cracker is a triangle!"
- "Let's make a chart that shows how many children like today's new snacks."

Dramatic play

- "You are using the big pot to cook."
- "This baby-doll is small. They need a smaller blanket."

Transitions

- "It's time to clean up. Will you help me put the toys inside the basket?"
- "Let's practice skip counting by twos as we line up."

Engaging in Math Discourse During a Meaningful Investigation

As the children explored their school garden, they excitedly compared their plants.

Child A grinned, pointing at their tall plant. "Mine's the biggest!"

Child B disagreed, stretching their arms out from side-to-side to indicate width. "But mine's bigger like this..."

The teacher asked, "How can we tell which plant is taller—from the bottom to the top? We might also find a way to see which one is wider—from this side to that side."

Child A used their hands as a ruler, counting, "One, two, three, four, five. Mine's five hands going up—tall. Let's check yours."

The teacher then offered string to measure. The children cut the string to match each plant's height.

"Now," the teacher asked, "how can we tell which plant is taller?"

Child A suggested, "Let's put the strings together and see if mine's longer. Mine's longer, so my plant's taller."

Child B added, "But mine's wider. Let's measure that too."

Repeat and extend. Scaffold children's language use by repeating what they say and, when appropriate, extending it. For example, if a child says "It's too big!" the educator might respond by saying "The ball of playdough is too big. There's too much playdough to fit inside the container."

This strategy validates children's ideas by letting them know they have been heard. Repeating and extending also allows educators to introduce new language and add greater specificity to children's ideas.

Use concrete objects or gestures. Use real objects and gestures when discussing math concepts. For example, move two tokens into a group of three tokens and say, "We can add two tokens to our group of three tokens to make one group of five tokens."

Gestures might also be used to communicate mathematical ideas. For example, an educator might say, "This one is long" and stretch their arms out wide. Or, placing one hand below the other, they might say, "What if we put it under?" Using concrete objects and gestures is especially helpful for multilingual learners. The strategy allows them to connect English words to familiar objects and actions.

Multiple Representations

What are multiple representations?

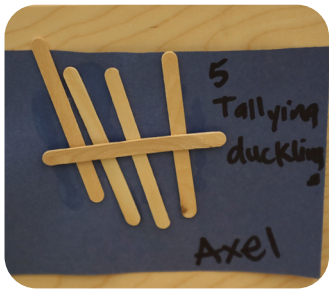
In mathematics, representation is a process of using models to organize, record, and communicate mathematical ideas.[40] Multiple representations provide opportunities for children to explore and express mathematical ideas in different ways. They include the use of various materials and learning aids such as pictures, models, tables, graphs, and manipulatives.

Multiple representations might also involve a variety of forms of engagement, such as movement, singing, touching, and building. For example, consider the different representations that children might use to add two numbers. They might write an equation (2 + 1 = 3), draw a picture (a group of two dots and a group of three dots), use a number line (starting at two, counting up one, and landing on three), count on their fingers, or move one more cotton ball to a group of two cotton balls and then count the total.

Why are multiple representations important?

Providing learners opportunities to use multiple representations of math ideas can enhance understanding and promote the transfer of knowledge and skills from one context to another. Multiple representations also allow learning experiences to better meet children's preferences, languages, cultures and lived experiences, abilities, and emerging knowledge and skills.

When children are exposed to multiple representations of mathematical concepts, they develop a deeper understanding of math.[41] They can observe how different representations are connected and how the same concept can be represented in different ways. For example, the number five might be represented using five objects, five tally marks, the numeral five, or five fingers. This broader understanding allows children to transfer their knowledge and skills across different situations. They can then apply what they learned in one setting or with one representation to solve problems or understand concepts in different settings or with different representations.



For example, children might use their knowledge of place value to represent numbers using base ten blocks—using a ten-rod and six unit-blocks to represent 16. Children with the ability to transfer their knowledge of place value might also represent numbers using expanded form: 16 = 10 + 6. By exploring multiple representations, children become better at noticing patterns within mathematical concepts. They can identify common structures and strategies that apply across different representations.[42]

Providing multiple representations for children supports the diverse learning needs and preferences of children. Some children may learn more effectively through visual representations (for example, using drawings), while others may grasp concepts better through concrete objects (for example, using counting chips).

Multiple representations can provide children, with and without disabilities, expanded opportunities to engage as math learners. For example, a child with visual impairments might benefit from using tactile dice with raised bumps to show quantity. Children who are nonspeaking might use gestures or visuals to communicate their ideas.

Using multiple representations can also engage children from diverse backgrounds and cultures. By providing multiple representations, educators create opportunities for children to demonstrate diverse ways of knowing that may be different from the way a textbook, curriculum, or educator represents an idea.[43] By encouraging children to use different ways to communicate and represent their math knowledge, educators can engage children with different learning preferences, experiences, strengths, and needs. This engagement thus enhances their overall learning experience.[44]

How to provide multiple representations

The strategies below offer ideas on how to promote the use of multiple representations in early math learning.

Provide access to, and model the use of, a variety of materials to express math ideas or represent math problems. Offer children access to math-specific materials (for example, ten-frames, base ten blocks, counting chips) and more general items (for example, loose parts and various art media such as clay, play dough, pom-pom balls).

Providing children access to such materials gives them opportunities to represent math ideas in different ways. Providing children access to various materials is especially helpful for children with diverse learning needs because they can select the materials that work best for their abilities.

Plan for ways to engage the whole body. When planning math experiences, consider how to involve children's whole bodies. Think about ways to integrate movement into learning experiences. For example, children might jump, hop, or clap as they count. Consider using songs or music to help children engage, construct meaning, or remember certain ideas. For example, simple songs like "Five Little Speckled Frogs" encourage children to think about subtraction. "Uno, Dos, Tres Amigos" introduces numbers in English and Spanish.

Provide open-ended learning experiences. Open-ended experiences allow children to use different approaches to solve math problems. These experiences might also have more than one solution. For example, educators might invite children to figure out how to create patterns with beads or how to figure out if more trains than buses are in their play area.

Open-ended experiences might also offer multiple entry points for children—they can work with the problem in a simple way as well as in a more complex way. These types of experiences encourage children with different math abilities to explore math concepts and allow children to express their understanding in various ways.

Conclusion

The M^5 approach is a comprehensive set of teaching practices that support math learning from birth into the early elementary years. M^5 includes five research-based, inquiry-driven, child-centered practices: mutual learning, meaningful investigations, materials and learning environment, math vocabulary and discourse, and multiple representations.

These practices are interconnected and complement each other. For example, mutual learning supports educators to offer meaningful investigations. Meaningful math investigations can include math-rich materials and environments that encourage the use of math vocabulary and discourse and offer multiple representations of math concepts.

Taken together, M^5 can help children have a strong sense of belonging in math learning, feel engaged and excited about math investigations, and build deep, meaningful understandings of math concepts.

Implementing the M^5 Early Math Approach

Start implementing M^5 in the learning setting. A first step toward embracing the M^5 approach is to notice how one is already using the practices. Then, one might consider ways to make changes to their practice over time—in a way that is comfortable and meaningful to themselves and the children in their learning setting.

Refer to the M^5 Early Math Approach Overview for brief descriptions of the M^5 practices. Use the Enhance My Practice planning sheet to think about ways to use M^5 practices in learning settings. For those facilitating professional learning sessions for early childhood educators (birth through the early elementary years), visit the Count Play Explore Professional Learning Suites for more support on sharing M^5 with educators.

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